

# DARK MATTER AND THE VOIDS

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## Abstract

It is now, generally, believed that the presence of some form of dark matter is essential to explain the flat rotation curves of galaxies, and anomalous large velocities of galaxies in the clusters and superclusters. This dark matter turns out to be too elusive and , so far, there is no direct evidence for such matter although there is a large variety of candidates proposed by theoretical models. One would expect that the existence of vast amount of dark matter would lead to the formation of some dark galaxies or clusters which could be seen by gravitational lensing events. This could serve as a direct evidence for dark matter. However, so far, there is no conclusive lensing event pertaining to that.

In spite of absence of any direct evidence, there is a consensus among physicists in favour of some form of non-baryonic dark matter. In this paper, we take a different viewpoint and would like to view dark matter as an **emergent** phenomenon - very much like many such phenomena known in condensed matter systems.

We propose here a novel mechanism involving Voids, which can, in principle give rise to a difference between the gravitational and the inertial mass, and thereby, create effects similar to the dark matter.

Basic forces of nature are characterized by the existence of fundamental constants. These constants not only characterize the distance scale of motion of bodies in force field, it also sets the size of the structure that can arise. Eletromagnetism probably gives rise to the largest variety of structures. This is because the scale in this case is set not just by the fine structure constant, but by the combination of fine structure constant  $\alpha$  and various atomic numbers  $Z$ . The strong interaction is largely responsible for the existence of elementary particles and nuclei. The weak interaction is responsible for the decay of such particle and, therefore, sets the stability standards. Gravitation, the the oldest known, interaction is not as well understood as the other three fundamental interactions. In the smallest scale we have the black hole singularity (distance of order of plank length  $\approx 10^{-33}$  cm.), in the intermediate scale the problem of dark matter and structure formation (distance scale ranging from a few kilopersec to a few megapersec) and in the largest scale (distance scale of around a few hundred megapersec), where we have the expansion of universe, there is the horizon problem and the problem of overall density, and also the possibility that the universe may be accelarating in the present epoch. In all these cases the constant that enters in the description is the Newton's constant of gravity,  $G = 6.67 \times 10^{-8} \text{ cm}^3/\text{gm}.\text{sec}^2$ . The ratio of the distance scales involved is approximately  $\approx 10^{28}/10^{-33} = 10^{61}$ , where as in the case of eletromagnetism it is roughly the ratio of the size of the Jupiter to the atomic size which is approximately  $\approx 10^{10}/10^{-8} = 10^{18}$ , in the case of strong interaction it is  $\approx 10^3$ . Keeping in mind that there is no quantum theory of gravity as yet, one wonders whether gravity is an effective theory with different coupling constants governing motions at different length scales (like the pion-nucleon coupling constant in strong interactions). After all, the direct verification of gravity extends in larger side of the scale only up to distances  $\approx 10^{19}$  cms. (a few parsecs in open clusters) and in smaller end of the scale not even submillimeter level. This is the enigma of gravity. In this paper, we will be concerned with only one of these problems. We leave aside the problem of the smallest and largest scale and consider only the intermediate scale ( from a few kilopersec to a few megapersec ). In this distance scale, we have the problem of the dark matter and the structure formation. **They both probably**

**are two sides of the same coin.**

The dark matter problem has been known for a long time. However, in seventies and early eighties when old data was analysed properly [1,2] and newly obtained data was analysed [3] that the problem of dark matter was well established. We expect that the velocities of the satellite galaxies and large gaseous clouds orbiting around the galaxies should fall off with distance as per the Coulomb law. However, what was found is that the velocities do not decrease at all. They rather become constant, and hence, the name flat rotation curves. It is also found that in galactic clusters the velocities of the galaxies are anomalously large and it is not possible to explain it by the gravitational field of the luminous matter alone. All sorts of exotic particles, distributed around the galaxies with density  $\rho(r) \sim 1/r^2$  have been postulated to explain the flat rotation curves. In these models, the amount of dark matter needed to explain the rotation curves is about eighty to ninety percent of total mass of the galaxies. Therefore, for the formation of large scale structure in the universe [4,5], they become the most relevant objects and dynamics would almost entirely be governed by them. All attempts to identify such exotic particles have been elusive. However, the search continues. There have also been search for the so called MACHO objects ( baryonic structures of the size of Jupiter) which could also explain the rotation curves. But this has also not produced satisfactory results. If the dark matter is non-baryonic, one would expect that the existence of vast amount of dark matter would lead to the formation of some dark galaxies or clusters which could be seen by gravitational lensing events. This could serve as a direct evidence for dark matter. However, so far, there is no conclusive lensing event pertaining to that. On the other hand, the absence of dark matter would imply that the theory of gravity be different at the intermediate scale. At this stage there is a serious psychological barrier based on very genuine reasons. After all, Newtonian gravity is the low velocity limit of the general theory of relativity. There may exist data not explained by these theories but there is no single data worth mentioning which directly contradicts them. Nevertheless, it does sound logical to construct models and look for their observational consequences. It is not enough to construct models which only explains the flat rotation

curves but, as mentioned before, it should also account for the large scale structure in the universe.

There have been some attempts to introduce such modifications. Sanders [6] suggested that Newton's constant of gravity varies with distance. The gravitational potential in his model is

$$U = -\frac{G(1 - \beta e^{-r/r_0})}{r(1 - \beta)} \quad (1)$$

The constants  $\beta$  and  $r_0$  can be determined from the data. However, there is no estimate for the size of structures or its role in large scale structure formation.

Milgrom and Bekenstein [7,8] assume that the ratio of inertial mass  $m_i$  and the gravitational mass  $m_g$  depends on acceleration in the small acceleration limit. To be more precise

$$m_i/m_g = \mu(a/a_o) \quad (2)$$

where  $a$  is the acceleration and  $a_o$  is a universal constant,  $a_o = 8h^{-2} \times 10^{-8} \text{ cm.sec}^{-2}$ .

$$\begin{aligned} \mu(a/a_o) &= 1 \text{ for } a \gg a_o \\ \mu(a/a_o) &= a \text{ for } a \ll a_o \end{aligned} \quad (3)$$

As a result of this assumption the law of gravity becomes very different when the potential gradient is small. Essentially, it means, that  $m_i a = m_g (GM/r^2)$  implies  $m_g a^2 = m_g (GM/r^2)$  in the small acceleration limit. Now, for a rotating body  $a = v^2/r$  where  $v$  is the rotational velocity. The above relations imply that  $v^4 = GM$ , and, therefore, the rotational velocity does not depend on distance. In this model, the kinetic energy is too soft and we do not expect large structure to emerge. This mechanism also fails to accommodate an well established observation that fainter (smaller) galaxies contain more dark matter.

Our approach, though different, accomodates some essential ideas of Milgrom and Bekenstein. In their approach, they try to modify the law of inertia for small accelerations. We,

instead, look for mechanism which can give rise to a difference between gravitational mass and the inertial mass. Our approach is in the spirit of condensed matter physics. It is in this sense that we view dark matter as an emergent phenomenon. We shall argue in this paper that Voids [9] can, in principle, create a difference between the gravitational and inertial mass. But before we come to that, the following discussion is necessary. It is well known that masses of particles can vary depending on the medium it is moving in. In condensed matter physics this is very well known. In fact, this is one of the basic ingredients in Landau Fermi liquid theory [10,11] . One often considers variation of mass as an artefact of quantum theory. But strictly speaking this is not correct - though quantum theory is the essential tool for carrying out analysis and quantitative calculations. The variation in masses, essentially reflects the nature of force field present in the medium. If the mass,  $m_B$ , ( $\mu = \frac{e\hbar}{2m_B c}$ ) , defining the magnetic moment of an electron, and the mass,  $m_F$ , ( $\epsilon_F = \frac{p^2}{2m_F}$ ), defining the Fermi energy, were always equal, there would not be any diamagnetic substance in nature. The very fact that there exist strong diamagnet such as bismuth, implies that in these materials the "two masses" are different,  $m_F < m_B$ . In such systems the inertial mass is lowered. There are other materials known as heavy fermion systems ( $CeAl_3$ ,  $CeCu_6$ ,  $UPt_3$  ), in which the inertial mass increases - the electrons acquire masses of several GeV. The inertial masses of particles are modified by the force field in the medium. Such a phenomenon is also known in classical Hydrodynamics [12]. The increase or decrease of inertial mass is determined by the nature of the drag the medium exerts on moving bodies - if the drag is positive the mass increases but if the drag is negative (as in the case of diamagnetic materials for electrons), the inertial mass decreases. It is this idea, which we want to carry over to the large structure. This requires review of some aspects of standard cosmology and the theory of structure formation at the large scale. This is briefly described below.

It is very well established through observation that at distance scales of the order of 200-300 megaparsec, the Universe is isotropic and homogenous. This means that if we pick up a region of the Universe of dimension 200-300 megaparsec at any distance and in any direction, it will contain the same amount of matter. Therefore, at this scale the density of

matter can be considered to be constant. **In Newtonian gravity, such a distribution of matter implies that at every point in space the potential and force are unbounded** [13] . This dilemma is resolved in the General Theory of Relativity. For an isotropic and homogenous distribution of matter one assumes the Friedman-Robertson-Walker metric, given by the line element

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (4)$$

in which the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{3} T_{\mu\nu} \quad (5)$$

takes the simple form [13],

$$\frac{\dot{a}^2 + ka^2}{a^4} = \frac{8\pi G}{3} \rho_0 \quad (6)$$

where  $a(t)$  is the scale factor,  $\rho_0$  is the averaged constant density, and  $k = 1, -1$  or  $0$ , respectively for closed, open and flat universe. This equation along with the equation of state describes the isotropic and homogenous universe. The equations clearly show that the isotropic and homogenous distribution of matter can not be stable - the Universe expands. The constant density serves as a source term in the evolution of the scale factor. What is the source of gravity in the large scale? When the mean free path of the particles is small, matter can be treated as an ideal fluid and the Newton's equations governing the motion of gravitating collisionless particles in an expanding Universe can be written in terms of  $\mathbf{x} = \mathbf{r}/a$  (the comoving space coordinate),  $\mathbf{v} = \dot{\mathbf{r}} - H\mathbf{r} = a\dot{\mathbf{x}}$  (the peculiar velocity field,  $H$  is the Hubble constant),  $\phi(\mathbf{x}, t)$  (the Newton gravitational potential) and  $\rho(\mathbf{x}, t)$  (the matter density). This give us the following set of equations [14,15]. Firstly, the Euler equation,

$$\frac{\partial(a\mathbf{v})}{\partial t} + (\mathbf{v} \cdot \nabla_{\mathbf{x}}) \mathbf{v} = -\frac{1}{\rho} \nabla_{\mathbf{x}} P - \nabla_{\mathbf{x}} \phi \quad (7)$$

Next, the continuity equation

$$\frac{\partial \rho}{\partial t} + 3H\rho + \frac{1}{a} \nabla_{\mathbf{x}}(\rho \mathbf{v}) = 0 \quad (8)$$

And, finally the Poisson equation

$$\nabla_{\mathbf{x}}^2 \phi = 4\pi G a^2 (\rho - \rho_0) = 4\pi G a^2 \rho_0 \delta \quad (9)$$

where  $\rho_0$  is the mean background density and  $\delta = \rho/\rho_0 - 1$  is the density contrast.

Therefore, at large scale, the source of gravity is not the average density  $\rho_0$  but the density fluctuations,  $\delta\rho > 0$ . It is a subject of study in theory of structure formation as to what kind of density fluctuation would grow in time and lead to the formation of galaxies, and clusters and superclusters of galaxies [4,5,14,15]. It is important, here, to remember that at the scale of dimensions, 200 - 300 megapersec, the Universe is homogenous and isotropic and acquires constant density and, therefore, if in some subregion  $\delta\rho > 0$ , there must be some subregion where  $\delta\rho < 0$ , so as to reproduce the constant density profile. These domains with  $\delta\rho < 0$  are known as **Voids** [9,14]. Note that for Voids  $\delta\rho/\rho_0$  is always bounded below by  $-1$ . Such regions of Voids dominate the volume in the universe giving rise to cellular structures with the clusters and superclusters of galaxies forming string like walls around them. Existence of Voids are supported by direct observation as well as numerical simulation of hydrodynamic equations [14,16–18]. The observed Voids seem to have dimension of several (tens of) megapersecs. However, Voids of our interest would be those whose dimensions are a few (tens of) kilopersec distributed within the intergalactic medium and within clusters and supercluetrs of galaxies. There is, so far, no observational evidence for Voids of such dimensions. However, issue of how the Voids are distributed is not yet settled. There are some numerical simmlations which suggest that Voids may form a connected network ( [14] and references there in). This connectivity of Void network is very important for cosmology. If the universe starts with a **matter coonnected network** and latter makes a transition to **Void connected network**, the rate of expansion will undergo abrupt change.

Let us suppose that Voids of such small dimensions exist and are distributed in the intergalactic medium and the clusters and superclusters of galaxies. The question that immediately arises is related to the observation of such objects. What are the observational

signature of such Voids? Large Voids are easily identified by the absence of galaxies and clusters in those regions. For smaller size Voids, possibly the ultra-low density limit of Saha ionization formula could be of some help because the Voids are ultra-low density regions. The formula implies that the atoms trapped in the Voids would prefer to remain ionized, and therefore, the random motion of ions and electrons would give rise to a faint radiation glow (see appendix for details).

Let us proceed with the assumption that Voids as discussed above exists. Since the source term responsible for the formation of Voids is the negative density contrast, it will give rise to force field that is opposite to that of gravity. Therefore, the intergalactic medium will be more like a **polarised region** - where Galaxies and Voids give rise to two opposite type of force fields. What effect would the Void have on a massive body? It would clearly exert a repulsive force at least locally. Therefore, a massive body moving around a galaxy or in the clusters would feel an extra push from the Voids. This effect can be taken into account by assuming that the inertial mass is effectively decreased. Thus the effect of Voids would bring about a difference in the "effective" inertial mass and the gravitational mass. This difference would appear as if there is some dark matter.

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## APPENDIX

### Ultra-low density limit of the Saha ionization formula

The Saha ionization formula [19] has played a very important role in the development of astrophysics. The ultra-low density limit of this formula has been known for a long time [20]. In this limit, the formula suggests that the atoms in equilibrium prefer to remain in ionized state. This ionization, just from "expansion" as the density goes down, has been listed as one of the surprises in theoretical physics by Peierls [21]. In this appendix, we point out that this ultra-low density limit of Saha ionization formula is very relevant for Voids.

The ionization formula is given by,

$$\frac{n_e n_i}{n_a} = \frac{1}{v_a} e^{-W/kT} \quad (10)$$

In the equation above,  $n_e$ ,  $n_i$ ,  $n_a$  are the densities of electrons, ions and atoms(not ionized) respectively.  $W$  is the ionization potential,  $T$  is the temperature and  $k$  is the Boltzmann constant. The volume occupied by a bound electron at temperature  $T$  is represented by  $v_a$ . It is, essentially, the volume contained within a thermal de Broglie wave length.

$$v_a = \lambda_{th}^3 = \left( \frac{2\pi\hbar^2}{m_e kT} \right)^{3/2} \quad (11)$$

Let us consider a box of volume  $V$  which, to start with, contains  $N$  number of hydrogen atoms. Let a fraction  $X$  of them be ionized. In this case,  $n_e = \frac{N}{V} X = n_i$  and  $n_a = (1 - X) \frac{N}{V}$ . Substituting these values in the ionization formula we obtain,

$$\frac{X^2}{1 - X} \frac{N}{V} = \frac{1}{v_a} e^{-W/kT} \quad (12)$$

From the equation above, we see that the fraction of charged particles in equilibrium increases when we increase the volume(i.e., decrease the density). In the ultra-low density ( $\frac{N}{V} \rightarrow 0$  or  $\frac{V}{N} \rightarrow \infty$ ), atoms would prefer to remain ionized.

The Voids are the ultra-low density regions in the Universe, and these are the regions where one would expect to observe the consequences of ultra-low density limit of the Saha ionization formula. As discussed before, in this limit, atoms would prefer to remain ionized.

Saha ionization formula implies that, at ultra-low density, once the ionization takes place there is hardly any chance for recombination [20–22]. Therefore, the random motion of the charged particles in the Voids should create a faint radiation glow.

At this stage, the important question is: what is the source of the ionization energy? One common source of ionization are the starlights. However, at high red shift, their intensity is very low. The most common source of ionization energy at high red shift are the lights from Quasars.

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